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260. Proposed by O. E. GLENN, Ph. D., Springfield, Mo.

The necessary and sufficient condition that a binary form be a perfect n th power is that its Hessian vanish.

I. Solution by G. W. GREENWOOD, M. A., McKendree College, Lebanon, Ill.

Denoting $\frac{\partial u}{\partial x}$ by p , $\frac{\partial u}{\partial y}$ by q , the vanishing of the Hessian shows that $p=f(q)$, i. e., $q=mp$, since both p and q are homogeneous and of the same degree. By Lagrange's method of solving partial differential equations, we have

$$\frac{dx}{m} = \frac{dy}{-1} = \frac{du}{0}.$$

Hence, $u=\text{constant}$, $x+my=\text{constant}$, and a general solution is given by

$$u=f(x+my)=(x+my)^n,$$

since u is homogeneous in x, y . It is easily verified that when $u=(x+my)^n$ the Hessian vanishes. Hence this condition is both necessary and sufficient.

II. Solution by the PROPOSER.

A slightly different point of view from the above is afforded by the following method:

The Hessian is the Jacobian of the first derivatives p and q . Hence $p-mq=0$. Also $xp+yq=nu$, n being the order of u . Solving for p and q ,

$$p=\frac{nm u}{y+mx}, \quad q=\frac{nu}{y+mx}.$$

$$\text{Also, } du=pdx+qdy=nu\frac{dy+mdx}{y+mx}, \quad \text{or } \frac{du}{u}=n\frac{d(y+mx)}{y+mx}.$$

Hence, $\log u=n \log k(y+mx)$, $u=(a_1x+a_2y)^n$.

261. Proposed by REV. R. D. CARMICHAEL, Hartselle, Ala.

Sum to infinity the series, $\frac{1}{n^p} + \frac{3}{n^{2p}} + \frac{5}{n^{3p}} + \frac{7}{n^{4p}} + \frac{9}{n^{5p}} + \dots$

Solution by G. W. GREENWOOD, M. A., McKendree College, Lebanon, Ill.

Denoting n^{-p} by x , we have

$$\begin{aligned} \sum_{i=1}^{\infty} \frac{(2i-1)}{n^{ip}} &= x[1+3x+5x^2+7x^3+\dots] \\ &= x \sum (2r+1)x^r = 2x \sum rx^r + x \sum x^r \end{aligned}$$

$$= 2x^2(1-x)^{-2} + x(1-x)^{-1} = x(1+x)(1-x)^{-2} = \frac{n^p+1}{(n^p-1)^2}$$

where we must have $|x| < 1$.

Also solved by Henry Heaton, A. H. Holmes, and G. B. M. Zerr.

CALCULUS.

217. Proposed by Professor F. ANDEREGG, Oberlin College, Oberlin, Ohio.

$$\text{Find } \lim_{n \rightarrow \infty} \frac{1}{n} \sqrt[n]{(n+1)(n+2)\dots(2n)}.$$

I. Solution by the PROPOSER.

$$\text{Let } x = \lim_{n \rightarrow \infty} \frac{1}{n} \sqrt[n]{(n+1)(n+2)\dots 2n}$$

$$= \lim_{n \rightarrow \infty} \sqrt[n]{\left(1 + \frac{1}{n}\right)\left(1 + \frac{2}{n}\right)\dots\left(1 + \frac{n}{n}\right)}.$$

$$\text{Then } \log x = \lim_{n \rightarrow \infty} \frac{1}{n} \log \left[\left(1 + \frac{1}{n}\right)\left(1 + \frac{2}{n}\right)\dots\left(1 + \frac{n}{n}\right) \right]$$

$$= \lim_{n \rightarrow \infty} \frac{1}{n} \sum_{\lambda=1}^{\lambda=n} \log \left(1 + \frac{\lambda}{n}\right) = \lim_{n \rightarrow \infty} \frac{1}{n} \sum_{\lambda=1}^{\lambda=n} \left(\frac{\lambda}{n} - \frac{\lambda^2}{2n^2} + \frac{\lambda^3}{3n^3} - \dots \right)$$

$$= \lim_{n \rightarrow \infty} \sum_{\lambda=1}^{\lambda=n} \sum_{\kappa=1}^{\kappa=n} (-1)^{\kappa-1} \frac{\lambda^{\kappa}}{\kappa n^{\kappa+1}}.$$

If the method of differences is used for $\sum_{\lambda=1}^{\lambda=n} \lambda^{\kappa} = 1^{\kappa} + 2^{\kappa} + 3^{\kappa} + \dots$, the κ th series of differences is

$$\begin{aligned} (\kappa+1)^{\kappa} - \binom{\kappa}{1} \kappa^{\kappa} + \binom{\kappa}{2} (\kappa-1)^{\kappa} - \binom{\kappa}{3} (\kappa-2)^{\kappa} + \dots \\ + (-1)^{\kappa-1} \binom{\kappa}{\kappa-1} 2^{\kappa} + (-1)^{\kappa} 1^{\kappa} \equiv \kappa!. \end{aligned}$$

The $(\kappa+1)$ th series is

$$(\kappa+2)^{\kappa} - \binom{\kappa+1}{1} (\kappa+1)^{\kappa} + \binom{\kappa+1}{2} \kappa^{\kappa} - \dots + (-1)^{\kappa} \binom{\kappa+1}{\kappa} 2^{\kappa} + (-1)^{\kappa+1} 1^{\kappa} \equiv 0,$$

κ being a positive integer.

If the first given number is represented by a and the successive differences by d_1, d_2, \dots

$$S_{n,\kappa} = \binom{n}{1} a + \binom{n}{2} d_1 + \binom{n}{3} d_2 + \dots + \binom{n}{\kappa+1} d_{\kappa}.$$